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Deconfinement at the Argyres-Douglas point in SU(2) gauge theory with broken $\mathcal{N}=2$ supersymmetry

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Abstract

We consider chiral condensates in SU(2) gauge theory with broken $\mathcal{N}=2$ supersymmetry. The matter sector contains an adjoint multiplet and one fundamental flavor. Matter and gaugino condensates are determined by integrating out the adjoint field. The only nonperturbative input is the Affleck-Dine-Seiberg (ADS) superpotential generated by one instanton plus the Konishi anomaly. These results are consistent with those obtained by the 'integrating in' procedure, including a reproduction of the Seiberg-Witten curve from the ADS superpotential. We then calculate monopole, dyon, and charge condensates using the Seiberg-Witten approach. We show that the monopole and charge condensates vanish at the Argyres-Douglas point where the monopole and charge vacua collide. We interpret this phenomenon as a deconfinement of electric and magnetic charges at the Argyres-Douglas point.

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1 Introduction

The derivation of exact results in $\mathcal{N}=1$ supersymmetric gauge theories based on low energy effective superpotentials and holomorphy was pioneered in [1,2], then new wave of development was initiated by Seiberg, see [3] for review. Additional input was provided by the Seiberg-Witten solution of $\mathcal{N}=2$ supersymmetric gauge theories with and without matter [4].

The key feature of the $\mathcal{N}=2$ theory is the existence of the Coulomb branch where the vacuum expectation value of the adjoint scalar serves as a modulus [4]. The solution is described in terms of Riemann surfaces and the Coulomb branch parametrizes the moduli space of their complex structures. The simplest way to break $\mathcal{N}=2$ supersymmetry (SUSY) down to $\mathcal{N}=1$ amounts to giving a nonvanishing mass μ to the chiral $\mathcal{N}=1$ superfield in the adjoint representation. This field is a partner to the gauge fields in the $\mathcal{N}=2$ supermultiplet. At small values of μ the theory is close to its $\mathcal{N}=2$ counterpart while at large μ the adjoint matter decouples and the pure $\mathcal{N}=1$ theory emerges. The emerging theory at large μ is close to supersymmetric QCD (SQCD) but does not coincide with it. A trace of the massive adjoint remains in the effective theory in the form of nonrenormalizable quartic terms [5] in the superpotential which are suppressed by $1/\mu$. Although in the $\mathcal{N}=1$ theory the degeneracy on the Coulomb branch is lifted by the superpotential, memory of the structure of the Riemann surfaces remains. Namely, the vanishing of the discriminant of the Riemann surface defines the set of vacua in the corresponding $\mathcal{N}=1$ theory [4–9].

Different vacua are distinguished by the values of chiral condensates, such as the gluino condensate $\langle \mathrm{Tr}\, \lambda\lambda \rangle$ and the condensate of fundamental matter $\langle \tilde{Q}Q \rangle$. Generically, the latter can be found in SQCD using the effective superpotential, while the gluino condensate can be evaluated using the Konishi anomaly which relates the two condensates (see Ref. [10] for a review). To obtain the condensate in pure $\mathcal{N}=1$ Yang-Mills theory one has to start with the massive SQCD and use the holomorphy to decouple massive matter. Recently, some points concerning formation of the condensate and identification of the relevant field configurations were clarified in [11–14]

The brane picture provides another approach to the problem. The brane configurations for $\mathcal{N}=1$ theories with different matter content are known [15] and the recipe for calculating minima of the superpotentials has been formulated [16]. The key point concerning the brane configurations is that to break SUSY down to $\mathcal{N}=1$ one has to rotate the $\mathcal{N}=2$ picture. However only configurations which correspond to the vanishing of the discriminant can be rotated which means that at any value of the adjoint mass these points remain intact. The superpotentials calculated from brane configurations are in correspondence with field theory expectations.

In this paper we consider an $\mathcal{N}=1$ theory with both adjoint and fundamental matter and limit ourselves to the most tractable case of SU(2) gauge group with one fundamental flavor and one multiplet in the adjoint representation. Our strategy is as follows: First, we integrate out the adjoint matter to get SQCD-like effective superpotential for the fundamental matter. The only nonperturbative input in this effective superpotential is given by the Affleck-Dine-Seiberg superpotential generated by one instanton [1]. Difference with pure SQCD is due to the tree level nonrenormalizable term generated by the heavy adjoint exchange, mentioned above. Similarly to SQCD, the effective superpotential together with the Konishi relations unambiguously fixes condensates of fundamental and adjoint matter as well as the gaugino condensates in all three vacua of the theory.

We then compare the condensate of the adjoint matter with points in the u plane corresponding to the vanishing of the discriminant defined by Seiberg-Witten solution in $\mathcal{N}=2$ theory and find a complete match. Our results for matter and gaugino condensates are consistent with those obtained by the 'integrating in' method [17, 18, 7] and can be viewed as an independent confirmation of this method. What is specific to our approach is that we start from the weak coupling regime where the notion of an effective Lagrangian is well defined, and then use holomorphy to extend results for chiral condensates into strong coupling.

We subsequently determine monopole, dyon, and charge condensates following the Seiberg-Witten approach, i.e. considering effective superpotentials near singularities on the Coulomb branch of the $\mathcal{N}=2$ theory. Again, holomorphy allows us to extend our results to the domain of the "hard" $\mathcal{N}=2$ breaking. This extension include not only the mass term of adjoint but also breaking of $\mathcal{N}=2$ in Yukawa couplings.

Our next step is the study chiral condensates in the Argyres-Douglas (AD) points. These points were originally introduced in the moduli/parameter space of $\mathcal{N}=2$ theories as points where two singularities on the Coulomb branch collide [19–21]. It is believed that the theory in the AD point flows in the infrared to a nontrivial superconformal theory. The notion of the AD point continues to make sense even when the $\mathcal{N}=2$ theory is broken to $\mathcal{N}=1$; in the $\mathcal{N}=1$ theory it is the point in parameter space where two vacua collide.

In particular, we consider the AD point where the monopole and charge vacua collide at a particular value of the mass of the fundamental flavor. Our key result is that both monopole and charge condensates vanish at the AD point. We interpret this as deconfinement of both electric and magnetic charges at the AD point.

Let us recall that the condensation of monopoles ensures confinement of quarks in the monopole vacuum [4], while the condensation of charges provides confinement of monopoles in the charge vacuum. As shown by 't Hooft [22] it is impossible for these two phenomena to coexist. This apparently leads to a paradoxical situation in the AD point where the monopole and charge vacua collide. Our result resolves this paradox.

This paradox is a part of more general problem: whether there is an uniquely defined theory at the AD point. Indeed, when two vacua collide the Witten index of the emerging effective theory at the AD point is fixed, namely there are two bosonic vacuum states. The question is whether there is any physical quantity which could serve as an order parameter differentiating these two vacua. The continuity of chiral condensates in the AD point we find shows that these condensates are not playing this role. The same continuity also leads to vanishing tension for domain walls interpolating between colliding vacua when we approach the AD point. We discuss if these domain walls could serve as a signal of two vacua in the AD point.

The paper is organized as follows. In Sec. 2 we dwell on the calculation of matter and gaugino condensates, while monopole, charge and dyon condensates are considered in Sec. 3. In Sec. 4 we briefly discuss a definition of the theory at the AD point and the related problem of domain walls. Our results are discussed in Sec. 5.

2 Matter and gaugino condensates

2.1 Effective superpotential and condensates

We consider a $\mathcal{N}=1$ theory with SU(2) gauge group where the matter sector consists of the adjoint field $\Phi^\alpha_\beta = \Phi^a(\tau^a/2)^\alpha_\beta$ ($\alpha,\beta = 1,2;\ a = 1,2,3$), and two fundamental fields Q^α_f ($f = 1,2$) describing one flavor. The general renormalizable superpotential for this theory has the form,

$$\mathcal{W} = \mu\,\mathrm{Tr}\,\Phi^2 + \frac{m}{2}\,Q^\alpha_f Q^f_\alpha + \frac{1}{\sqrt{2}}\,h^{fg}\,Q_{\alpha f}\Phi^\alpha_\beta Q^\beta_g\,. \tag{1}$$

Here the parameters μ and m are related to the masses of the adjoint and fundamental fields, $m_\Phi = \mu/Z_\Phi$, $m_Q = m/Z_Q$, by the corresponding Z factors in the kinetic terms. Having in mind normalization appropriate for the $\mathcal{N}=2$ case we choose for bare parameters $Z^0_\Phi = 1/g_0^2$, $Z^0_Q = 1$. The matrix of Yukawa couplings h^{fg} is symmetric, and summation over color indices $\alpha,\beta = 1,2$ is explicit. Unbroken $\mathcal{N}=2$ SUSY appears when $\mu = 0$ and $\det h = -1$.

To obtain an effective theory similar to SQCD we integrate out the adjoint field Φ implying that $m_\Phi \gg m_Q$. In the classical approximation this integration reduces to the substitution

$$\Phi^\alpha_\beta = -\frac{1}{2\sqrt{2}\,\mu}\,h^{fg}\left(Q_{\beta f}Q^\alpha_g - \frac{1}{2}\,\delta^\alpha_\beta\,Q_{\gamma f}Q^\gamma_g\right)\,, \tag{2}$$

which follows from $\partial\mathcal{W}/\partial\Phi = 0$. What is the effect of quantum corrections on the effective superpotential? It is well known from the study of SQCD that perturbative loops do not contribute and nonperturbative effects are exhausted by the Affleck-Dine-Seiberg (ADS) superpotential generated by one instanton [1]. The effective superpotential then is

$$\mathcal{W}_{\rm eff} = m\,V - \frac{(-\det h)}{4\mu}\,V^2 + \frac{\mu^2\Lambda_1^3}{4\,V} \tag{3}$$

where the gauge and subflavor invariant chiral field V is defined as

$$V = \frac{1}{2}\,Q^\alpha_f Q^f_\alpha\,. \tag{4}$$

The first two terms in Eq. (3) appear on the tree level after substitution (2) into Eq. (1) while the third nonperturbative one is the ADS superpotential. The scale parameter Λ_1 is given in terms of the mass of Pauli-Villars regulator $M_{\rm PV}$ and the bare coupling g_0 (plus the vacuum angle θ_0) as

$$\Lambda_1^3 = 4\, M_{\rm PV}^3 \exp\left(-\frac{8\pi^2}{g_0^2} + i\theta_0\right) \; . \tag{5}$$

The coefficient $\mu^2\Lambda_1^3/4$ in the ADS superpotential is equivalent to $\Lambda_{\rm SQCD}^5$ in SQCD. The factor μ^2 in the coefficient reflects four fermionic zero modes of the adjoint field, see e.g. Ref. [23, 13] for details.

The only term in the superpotential (3) which differentiates it from the SQCD case is the second term which is due to tree level exchange by the adjoint field. At $h = 0$ it vanishes and we are back to the known SQCD case with two vacua and a Higgs phase for small m.

When $\det h$ is nonvanishing we have three vacua, marked by the vevs of the lowest component of V,

$$v = \langle\, V \,\rangle \, . \tag{6}$$

These vevs are roots of the algebraic equation $\mathrm{d}\mathcal{W}_{\rm eff}/\mathrm{d}v = 0$ which has the form

$$m - \frac{(-\det h)}{2}\,\frac{v}{\mu} - \frac{\Lambda_1^3}{4}\left(\frac{\mu}{v}\right)^2 = 0 \; . \tag{7}$$

This equation shows, in particular, that although the second term in the superpotential (3) seems to be suppressed at large μ it turns out to be of the same order as the ADS term. From Eq. (7) it is also clear that the dependence on μ is given by the scaling $v \propto \mu$.

To see the dependence on the other parameters let us substitute v by the dimensionless variable κ defined by the relation

$$v = \mu \sqrt{\frac{\Lambda_1^3}{4m}}\, \kappa \; . \tag{8}$$

Then Eq. (7), when rewritten in terms of κ,

$$1 - \sigma\,\kappa - \frac{1}{\kappa^2} = 0 \tag{9}$$

is governed by the dimensionless parameter σ,

$$\sigma = \frac{(-\det h)}{4}\left(\frac{\Lambda_1}{m}\right)^{3/2} . \tag{10}$$

We see that the two parameters m and $\det h$ enter only as $m\,(-\det h)^{-2/3}$. The dependence of v on μ is linear as we discussed above.

The particular dependence of condensate v on the parameters μ, m and $\det h$ follows from the R symmetries of the theory. Following Seiberg [24] one can consider μ, m and $\det h$ as background fields and identify nonanomalous R symmetries which prove the dependence discussed above. Classically, there are three U(1) symmetries in the theory associated with the three fermion fields (gaugino, adjoint and fundamental fermions). In the quantum theory one can organize two nonanomalous combinations (a symmetry is nonanomalous if it does not transform the scale Λ_1, associated with regulators).

The charges of the fields and parameters of the theory under these two U(1) symmetries are shown in Table 1. The first of these symmetries $\mathrm{U}_J(1)$ is a subgroup of

Fields/parameters	Φ	Q	W	θ	m	μ	h
$\mathrm{U}_J(1)$ charges	0	1	1	1	0	2	0
$\mathrm{U}_R(1)$ charges	1	-1	1	1	4	0	3

Table 1: Nonanomalous U(1) symmetries

the global $\mathrm{SU}_R(2)$ group related to the $\mathcal{N}=2$ superalgebra [4]. This explains the zero charge of the coupling h with respect to this symmetry. The symmetry $\mathrm{U}_J(1)$ fixes the μ dependence of condensates. Namely, it is given by a power of μ equal to half the $\mathrm{U}_J(1)$ charge of the condensate. In particular, the field V has $\mathrm{U}_J(1)$ charge equal to 2 which ensures that $v \propto \mu$. Thus, we can use holomorphy to extend results to arbitrary values of μ.

The second nonanomalous symmetry $\mathrm{U}_R(1)$ is similar to the R symmetry of Ref. [1] extended to include the adjoint field. As a consequence, for a given chiral field X

$$\langle X \rangle = \mu^{Q_J/2} m^{Q_R/4} \Lambda_1^{d_X-(Q_J/2)-(Q_R/4)} f_X(\sigma)\,, \tag{11}$$

where Q_J, Q_R are the $\mathrm{U}_J(1)$, $\mathrm{U}_R(1)$ charges of the field X, d_X is its dimension, and f_X is an arbitrary function of the dimensionless parameter σ defined by Eq. (10). This parameter is neutral under both U(1)'s. The equation (8) is an example of the general relation (11) with $Q_J=2$, $Q_R=-2$ and $f_V=\kappa(\sigma)/2$.

The important benefit of the consideration above is that in a theory with $\mathcal{N}=2$ SUSY strongly broken by large μ and $\det h \neq -1$ we can still relate chiral condensates with those in softly broken $\mathcal{N}=2$ where $\det h=-1$ and μ is small.

Here is an example. When $\sigma \to 0$ two roots of Eq. (9) are $\kappa_{1,2}=\pm 1$ and the third one goes to infinity as $\kappa_3 = 1/\sigma$. For two finite roots one can suggest dual interpretations. Firstly, taking $h=0$, one can relate them to two vacua of SQCD in the Higgs phase. Second, for $\det h=-1$ (which is its $\mathcal{N}=2$ value) one can make σ small by taking the limit of large m. But this limit should bring us to the monopole and dyon vacua of softly broken $\mathcal{N}=2$ SYM. The naming of vacua refers to the particle whose mass vanishes in the corresponding vacuum.

To verify this interesting mapping we need to determine the vev

$$u = \langle U \rangle = \langle \mathrm{Tr}\, \Phi^2 \rangle \,, \tag{12}$$

which can be accomplished using the set of Konishi anomalies. Generic equation for an arbitrary matter field Q looks as follows (we are using the notation of the review [10]):

$$\frac{1}{4}\, \bar{D}^2 J_Q = Q\, \frac{\partial \mathcal{W}}{\partial Q} + T(R)\, \frac{\mathrm{Tr}\, W^2}{8\pi^2} \,, \tag{13}$$

where $T(R)$ is the Casimir in the matter representation. The left hand side is a total derivative in superspace so its average over any supersymmetric vacuum vanishes. In our case this results in two relations for the condensates,

$$\begin{aligned} &\left\langle \frac{m}{2}\, Q^\alpha_f Q^f_\alpha + \frac{1}{\sqrt{2}}\, h^{fg}\, Q_{\alpha f} \Phi^\alpha_\beta Q^\beta_g + \frac{1}{2}\, \frac{\mathrm{Tr}\, W^2}{8\pi^2} \right\rangle = 0 \\ &\left\langle 2\, \mu \mathrm{Tr}\, \Phi^2 + \frac{1}{\sqrt{2}}\, h^{fg}\, Q_{\alpha f} \Phi^\alpha_\beta Q^\beta_g + 2\, \frac{\mathrm{Tr}\, W^2}{8\pi^2} \right\rangle = 0 \end{aligned} \tag{14}$$

From the first relation, after the substitution in (2) and comparing with Eq. (7), we find an expression for gluino condensate [25]

$$s = \frac{\langle \mathrm{Tr}\, \lambda^2 \rangle}{16\pi^2} = -\frac{\langle \mathrm{Tr}\, W^2 \rangle}{16\pi^2} = \frac{\mu^2 \Lambda_1^3}{4\, v} \,. \tag{15}$$

This is consistent with the general expression $[T_G - \sum T(R)]\langle \mathrm{Tr}\lambda^2 \rangle / 16\pi^2$ for the non-perturbative ADS piece of the superpotential (3), see [26]. Combining the two relations in (14) we can express the condensate u in terms of v,

$$u = \frac{1}{2\mu}\, (m\, v + 3\, s) = \frac{1}{2\mu} \left(m\, v + \frac{3}{4}\, \frac{\mu^2 \Lambda_1^3}{v} \right) = \frac{\sqrt{m \Lambda_1^3}}{4} \left(\kappa + \frac{3}{\kappa} \right) \,. \tag{16}$$

Now we see that in the limit of large m two vacua $\kappa = \pm 1$ are in perfect correspondence with $u = \pm\, \Lambda_0^2$ for the monopole and dyon vacua of $\mathcal{N}=2$ SYM. Indeed, $\Lambda_0^4 = m \Lambda_1^3$ is the correct relation between the scale parameters of the theories.

For the third vacuum at large m the value $u = m^2/(-\det h)$ corresponds on the Coulomb branch to the so called charge vacuum, where some fundamental fields become massless. Moreover, the correspondence with $\mathcal{N}=2$ results can be demonstrated for the three vacua at any value of m. To this end we use the relation (16) and Eq. (9) to derive the following equation for u,

$$(-\det h)\, u^3 - m^2\, u^2 - \frac{9}{8}\, (-\det h)\, m \Lambda_1^3\, u + m^3 \Lambda_1^3 + \frac{27}{2^8}\, (-\det h)^2 \Lambda_1^3 = 0 \,. \tag{17}$$

The three roots of this equation are the vevs of Tr Φ^2 in the corresponding vacua.

How does this look from $\mathcal{N}=2$ side? The Riemann surface governing the Seiberg-Witten solution is given by the curve [4]

$$y^2 = x^3 - u\,x^2 + \frac{1}{4}\Lambda_1^3 m\,x - \frac{1}{64}\Lambda_1^6\,. \tag{18}$$

Singularities of the metric, i.e. the points in the u-plane where the discriminant of the curve vanishes are defined by two equations, $y^2 = 0$ and $\mathrm{d}y^2/\mathrm{d}x = 0$,

$$x^3 - u\,x^2 + \frac{1}{4}\Lambda_1^3 m\,x - \frac{1}{64}\Lambda_1^6 = 0\,, \quad 3x^2 - 2u\,x + \frac{1}{4}\Lambda_1^3 m = 0\,, \tag{19}$$

which lead to

$$u^3 - m^2\,u^2 - \frac{9}{8}\,m\Lambda_1^3\,u + m^3\Lambda_1^3 + \frac{27}{2^8}\,\Lambda_1^3 = 0\,. \tag{20}$$

We see that this is a particular case of the $\mathcal{N}=1$ equation (17) at $\det h = -1$.

Moreover, when $\det h$ is not equal to its $\mathcal{N}=2$ value (-1) Eq. (17) coincides with Eq. (20) after the rescaling

$$u = (-\det h)^{1/3}\,u'\,, \quad m = (-\det h)^{2/3}\,m'\,, \quad v = (-\det h)^{-1/3}\,v'\,. \tag{21}$$

This is in agreement with the master parameter σ which contains the product $m^{-3/2}\det h$ and the nonanomalous U(1) symmetries we discussed above. In other words, breaking of $\mathcal{N}=2$ by Yukawa couplings does not influence consideration of the chiral condensates modulus the rescaling (21).

The consideration above shows that the only nonperturbative input needed to determine the chiral condensates is provided by the one-instanton ADS superpotential. This means that any reference to the $\mathcal{N}=2$ limit is not crucial at all, i.e. in regard to these condensates the exact Seiberg-Witten solution of $\mathcal{N}=2$ is equivalent to the ADS superpotential.

The relations for the condensates we have derived are not new, they were obtained in [7] by the 'integrating in' procedure introduced in [18]. Our approach which is based on 'integrating out', plus the Konishi relations, can be viewed as an independent proof of the 'integrating in' procedure.

What we see as an advantage of our approach it is that, within a certain range of parameters, the superpotential (3) gives a complete description of the low energy physics. Indeed, when the mass m_V of the field V,

$$m_V = 2m\,(2 - 3\sigma\kappa)\,, \tag{22}$$

is much less than the other masses, such as $m_\Phi = g^2\mu$ and $m_W = |g^2 v|^{1/2}$, we are in the weakly coupled Higgs phase and enjoy full theoretical control. The Konishi relations help to determine the condensates of heavy fields in this phase. Holomorphy then allows for continuation of these results for the condensates to strong coupling.

At strong coupling the superpotential (3), like other versions of the Veneziano-Yankielowicz Lagrangians [27], does not describe the low energy physics. For example, it contains no light monopole degrees of freedom near the monopole vacuum point at small μ. Moreover, there is no single local superpotential which could describe mutually nonlocal degrees of freedom which become light in different regions of the moduli space of the theory. At strong coupling the superpotential (3) is equivalent to the effective superpotential [7] of the 'integrating in' procedure and can be viewed as a shorthand equation that gives the values of the condensates.

2.2 Matter and gaugino condensates in the limit of large mass

Here we summarize the results for matter, $v = \langle Q^\alpha_f Q^f_\alpha/2\rangle$, $u = \langle \mathrm{Tr}\ \Phi^2\rangle$, and gaugino, $s = \langle \mathrm{Tr}\ \lambda^2/16\pi^2\rangle$, condensates in the limit where the parameter σ defined by Eq. (10) is small. This can be achieved in the limit of large m if the Yukawa coupling is fixed, or by taking $\det h$ to be small otherwise. In the charge vacuum:

$$\begin{aligned} v_C &= \frac{2\,\mu\, m}{(-\det h)} \cdot \Big(1 + \mathcal{O}(\sigma^2)\Big)\,, \\ u_C &= \frac{m^2}{(-\det h)} \cdot \Big(1 + \mathcal{O}(\sigma^2)\Big)\,, \\ s_C &= \frac{\mu\, \Lambda_1^3 (-\det h)}{8\, m} \cdot \Big(1 + \mathcal{O}(\sigma^2)\Big)\,. \end{aligned} \tag{23}$$

In the monopole and dyon vacua:

$$\begin{aligned} v_{\,M,D} &= \pm\, \mu \sqrt{\frac{\Lambda_1^3}{4m}} \cdot (1 + \mathcal{O}(\sigma))\,, \\ u_{\,M,D} &= \pm \sqrt{\Lambda_1^3 m} \cdot (1 + \mathcal{O}(\sigma))\,, \\ s_{\,M,D} &= \pm\, \frac{1}{2}\, \mu \sqrt{\Lambda_1^3 m} \cdot (1 + \mathcal{O}(\sigma))\,. \end{aligned} \tag{24}$$

The upper sign refers to the monopole vacuum, while the lower one is for the dyon vacuum. As discussed above we can interpret these vacua also as the two vacua of the Higgs phase in SQCD. To this end we need to consider the limit of small $\det h$ and $m \ll \Lambda_{\mathrm{SQCD}}$ with the identification

$$\Lambda^5_{\mathrm{SQCD}} = \frac{1}{4}\, \mu^2 \Lambda_1^3 \tag{25}$$

2.3 Small mass limit

The limit of massless fundamentals $m \to 0$ corresponds to $\sigma \to \infty$. In this limit the three vacua are related by a Z_3 symmetry [4],

$$v = \frac{\mu\,\Lambda_1}{(2\det h)^{1/3}}\, e^{2\pi i k/3} \cdot \Big(1 + \mathcal{O}(\sigma^{-2/3})\Big)\,, \qquad (k=0,\pm1),$$
$$u = \frac{3}{8}\,\Lambda_1^2\,(2\det h)^{1/3}\;\, e^{-2\pi i k/3} \cdot \Big(1 + \mathcal{O}(\sigma^{-2/3})\Big)\,, \qquad (k=0,\pm1),$$
$$s = \frac{1}{4}\,\mu\,\Lambda_1^2\,(2\det h)^{1/3}\;\, e^{-2\pi i k/3} \cdot \Big(1 + \mathcal{O}(\sigma^{-2/3})\Big)\,, \qquad (k=0,\pm1) \qquad (26)$$

Note that the massless limit exists due to the nonvanishing Yukawa coupling. When $h \to 0$ we are back to the runaway vacua of massless SQCD.

2.4 Argyres-Douglas points

When the mass m changes from large to small values we interpolate between the two quite different structures of vacua shown above. Let us consider this transition when, for definiteness, $\det h = -1$ and m is real and positive and changes from large to small values. At large positive m all the vacua are situated at real values of u, from Eqs. (23, 24) we see that the dyon vacuum is at negative u, the monopole vacuum is at positive u, and the charge vacuum is also at positive, but much larger, values of u. When m diminishes then at some point the monopole and charge vacua collide on the real axis of u and subsequently go more off to complex values producing the Z_3 picture at small m.

The point in the parameter manifold where the two vacua coincide is the AD point [19]. In the SU(2) theory these points were studied in [20]. Mutually non-local states, say charges and monopoles, becomes massless at these points. On the Coulomb branch of the $\mathcal{N}=2$ theory these points correspond to a non-trivial conformal field theory [20]. Here we study the $\mathcal{N}=1$ SUSY theory, where $\mathcal{N}=2$ is broken by the mass term for the adjoint matter as well as by the difference of the Yukawa coupling from its $\mathcal{N}=2$ value. Collisions of two vacua still occur in this theory. In this subsection we find the values of m at which AD points appear and calculate the values of the condensates at this point. In the next section we study what happens to the confinement of charges in the monopole point at non-zero μ once we approach the AD point.

First, let us work out the AD values of m, generalizing the consideration in [20]. Collision of two roots for v means that together with Eq. (7) the derivative of its left-hand-side should also vanish,

$$m - \frac{(-\det h)}{2}\,\frac{v}{\mu} - \frac{\Lambda_1^3}{4}\left(\frac{\mu}{v}\right)^2 = 0, \qquad -(-\det h) + \Lambda_1^3\left(\frac{\mu}{v}\right)^3 = 0\,. \qquad (27)$$

This system is consistent only at three values of $m = m_{\rm AD}$,

$$m_{\rm AD} = \frac{3}{4}\,\omega\,\Lambda_1\,(-\det h)^{2/3}, \qquad \omega = \mathrm{e}^{2\pi i n/3} \quad (n=0,\pm1)\,, \qquad (28)$$

related by Z_3 symmetry. The condensates at the AD vacuum are

$$v_{\rm AD} = \omega \, \frac{\mu \, \Lambda_1}{(-\det h)^{1/3}} \,,$$
$$u_{\rm AD} = \omega^{-1} \, \frac{3}{4} \, \Lambda_1^2 \, (-\det h)^{1/3} \,,$$
$$s_{\rm AD} = \omega^{-1} \, \frac{1}{4} \, \mu\Lambda_1^2 \, (-\det h)^{1/3} \,. \tag{29}$$

3 Dyon condensates

In this section we calculate various dyon condensates at the three vacua of the theory. As discussed above, holomorphy allows us to find these condensates starting from a consideration on the Coulomb branch in $\mathcal{N}=2$ near the singularities associated with a given massless dyon. Namely, we calculate the monopole condensate near the monopole point, the charge condensate near the charge point and the dyon $(n_m, n_e) = (1,1)$ condensate near the point where this dyon is light. Although we start with small values of the adjoint mass parameter μ, our results for condensates are exact for any μ as well as for any value of $\det h$.

3.1 Monopole condensate.

Let us start with calculation of the monopole condensate near the monopole point. Near this point the effective low energy description of our theory can be given in terms of $\mathcal{N}=2$ dual QED [4]. It includes a light monopole hypermultiplet interacting with a vector (dual) photon multiplet in the same way as electric charges interact with ordinary photons. Following Seiberg and Witten [4] we write down the effective superpotential in the following form,

$$W = \sqrt{2} \, \tilde{M} M A_D + \mu \, U, \tag{30}$$

where A_D is a neutral chiral field (it is a part of the $\mathcal{N}=2$ dual photon multiplet in the $\mathcal{N}=2$ theory) and $U = \mathrm{Tr}\, \Phi^2$ considered as a function of A_D. The second term breaks $\mathcal{N}=2$ supersymmetry down to $\mathcal{N}=1$.

Varying this superpotential with respect to A_D, M and $\tilde{M}$ we find that $A_D = 0$, i.e. the monopole mass vanishes, and

$$\langle \tilde{M} M \rangle = - \, \frac{\mu}{\sqrt{2}} \frac{{\rm d}u}{{\rm d}a_D}\bigg|_{a_D=0} \,. \tag{31}$$

The condition $A_D = 0$ means that the Coulomb branch near the monopole point, where the monopole mass vanishes, shrinks to a single vacuum state at the singularity while Eq. (31) determines the value of monopole condensate. Below we consider a_D as a function of u. The value of u, $u = u_M$, at the monopole vacuum was determined in the previous section.

The non-zero value of the monopole condensate ensures U(1) confinement for charges via the formation of Abrikosov-Nielsen-Olesen vortices. Let us work out the r.h.s. of Eq. (31) to determine the μ and m dependence of the monopole condensate. From the exact Seiberg-Witten solution [4], we have

$$\frac{\mathrm{d}a_D}{\mathrm{d}u} = \frac{\sqrt{2}}{8\pi} \oint_\gamma \frac{\mathrm{d}x}{y(x)}\,. \tag{32}$$

Here for $y(x)$ given by Eq. (18) we use the form

$$y^2 = (x - e_0)(x - e_-)(x - e_+)\,. \tag{33}$$

The integration contour γ in the x plane circles around two branch points e_+ and e_- of $y(x)$. At the monopole vacuum, when $u = u_M$, two branch points e_+ and e_- coincide, $e_+ = e_- = e$ and the integral (32) is given by the residue at $x = e$.

$$\frac{\mathrm{d}a_D}{\mathrm{d}u}(u_M) = \frac{i\,\sqrt{2}}{4\,\sqrt{e - e_0}}\,. \tag{34}$$

The value of $e - e_0$ (equal at $u = u_M$ to $(1/2)\,\mathrm{d}^2(y^2)/\mathrm{d}x^2$) is fixed by the equation $\mathrm{d}(y^2)/\mathrm{d}x = 0$,

$$e - e_0 = \sqrt{u_M^2 - \frac{3}{4} m \Lambda_1^3}\,. \tag{35}$$

Substituting this into the expression for the monopole condensate (31) we get finally

$$\langle \tilde{M} M \rangle = 2 i \mu \left(u_M^2 - \frac{3}{4} m \Lambda_1^3 \right)^{1/4}. \tag{36}$$

To test the result let us consider first the limit of a large masses m for the fundamental matter. As in Sec. 2.1 this limit can be viewed as a RG flow to pure Yang-Mills theory with the identification

$$\Lambda_0^4 = m \Lambda_1^3, \tag{37}$$

where Λ_0 is the scale of the $\mathcal{N}$=2 Yang-Mills theory. In this limit we have $u_M = \Lambda_0^2$. Then Eq. (36) gives

$$\langle \tilde{M} M \rangle = \sqrt{2}\, i\, \mu\, \Lambda_0\,, \tag{38}$$

which coincides with the Seiberg-Witten result [4]. This ensures monopole condensation and charge confinement in the monopole point at large m.

Notice, that in the derivation above $\mathcal{N}$=2 was not broken by the Yukawa coupling, i.e. we assume $\det h = -1$. The result, however, can be easily generalized to arbitrary $\det h$ by means of U(1) symmetries considered above. The U(1) charges of $\tilde{M} M$ coincide with those of V. Taking this into account we get for the monopole condensate at $\det h \neq -1$

$$\langle \tilde{M} M \rangle = \frac{2i\mu}{\sqrt{-\det h}} \left(u_M^2 - \frac{3}{4} m \Lambda_1^3 \right)^{1/4} . \tag{39}$$

The result is consistent also with the rescaling (21). This means, in particular, that $\mathcal{N}=2$ breaking in the Yukawa coupling h in the microscopic theory leads to $\mathcal{N}=2$ breaking in the Yukawa coupling of monopoles in the dual theory. Namely, the first term in Eq. (30) acquires the factor $\sqrt{-\det h}$.

It is instructive to rewrite the result (36) for the monopole condensate in terms of v,

$$\langle \tilde{M} M \rangle^2 = -\left[v^2 - \frac{\mu^3 \Lambda_1^3}{v(-\det h)} \right] = -\left[v^2 - \frac{4\mu\, s}{(-\det h)} \right] , \tag{40}$$

where we also show a form which uses the gluino condensate s. These results follow from the expression (16) for u and Eq. (7) for v. It is interesting to observe that when $v \gg \mu\Lambda_1$ the nonperturbative term can be neglected and the monopole condensate reduces to that of the quark, $\langle \tilde{M} M \rangle \to iv$. We will return to a discussion of this relation in the next subsection where other condensates are found.

It is interesting to note also, that in the SQCD limit of small $\det h$ the monopole condensate given by Eq. (40) becomes parametrically large.

Now let us address the question: what happens with the monopole condensate when we reduce m and approach the AD point? The AD point corresponds to a particular value of m which ensures collision of the monopole and charge singularities in the u plane. Near the monopole point we have condensation of monopoles and confinement of charges while near the charge point we have condensation of charges and confinement of monopoles. As shown by 't Hooft these two phenomena cannot happen simultaneously [22]. The question is: what happens when monopole and charge points collide in the u plane?

The monopole condensate at the AD point is given by Eq. (39). When m and u are substituted by m_{AD} and u_{AD} from Eqs. (28) and (29), we get

$$\langle \tilde{M} M \rangle_{AD} = 0. \tag{41}$$

We see that the monopole condensate goes to zero at the AD point. Our derivation makes it clear why it happens. At the AD point all three roots of y^2 become degenerate, $e_+ = e_- = e_0$, so the monopole condensate which is proportional to $\sqrt{e - e_0}$ naturally vanishes.

In the next subsection we calculate the charge condensate in the charge point and show that it also goes to zero as m approaches its AD value (28). Thus we interpret the AD point as a deconfinement point for both monopoles and charges.

3.2 Charge and dyon condensates

In this subsection we use the same method to calculate values for the charge and dyon condensates near the charge and dyon points respectively. We first consider m above its AD value (28) and then continue our results to values of m below m_{AD}. In particular, in the limit $m = 0$ we recover Z_3 symmetry.

Let us start with the charge condensate. At $\mu = 0$, $\det h = -1$ and large m the effective theory near the charge point

$$a = -\sqrt{2}\, m \tag{42}$$

on the Coulomb branch is $\mathcal{N}=2$ QED. Here a is the neutral scalar, the partner of photon in the $\mathcal{N}=2$ supermultiplet. Half the degrees of freedom in color doublets become massless whereas the other half acquire large a mass $2m$. The massless fields form one hypermultiplet $\tilde{Q}_+, Q_+$ of charged particles in the effective electrodynamics. Once we add the mass term for the adjoint matter the effective superpotential near the charge point becomes

$$\mathcal{W} = \frac{1}{\sqrt{2}}\, \tilde{Q}_+ Q_+ A + m\, \tilde{Q}_+ Q_+ + \mu\, U \tag{43}$$

Minimizing this superpotential we get condition (42) as well as

$$\langle \tilde{Q}_+ Q_+ \rangle = -\, \sqrt{2}\, \mu\, \frac{\mathrm{d}u}{\mathrm{d}a}\bigg|_{a=-\sqrt{2}\, m} . \tag{44}$$

Now, following the same steps which led us from (31) to (36), we get

$$\langle \tilde{Q}_+ Q_+ \rangle = \frac{2\,\mu}{\sqrt{-\det h}} (u_C^2 - \frac{3}{4} m\, \Lambda_1^3)^{1/4} , \tag{45}$$

where we include a generalization to arbitrary $\det h$. Here u_C is the position of the charge point in the u plane. At large m $u_C = m^2/(-\det h)$, see Eq. (23), and

$$\langle \tilde{Q}_+ Q_+ \rangle = \frac{2\,\mu\, m}{(-\det h)} \Big(1 + \mathcal{O}(\sigma^2)\Big) . \tag{46}$$

Holomorphy allows us to extend the result (45) to arbitrary m and $\det h$. So we can use Eq. (45) to find the charge condensate at the AD point. Using Eqs. (28) and (29) we see that the charge condensate vanishes at the AD point in the same manner the monopole condensate does. As it was mentioned we interpret this as deconfinement for both charges and monopoles.

As with the monopole condensate, we can also relate the charge condensate with the quark vev v,

$$\langle \tilde{Q}_+ Q_+ \rangle^2 = v^2 - \frac{\mu^3 \Lambda_1^3}{v(-\det h)} = v^2 - \frac{4\mu\, s}{(-\det h)}, \tag{47}$$

This expression differs from the one for the monopole condensate only by a sign. The coincidence of the charge condensate with the quark one at large v, i.e. at weak coupling, is natural. The difference is due to nonperturbative effects and is similar to the difference between $a^2/2$ and u on the Coulomb branch of the $\mathcal{N}=2$ theory. At strong coupling the difference is not small. In particular, the charge condensate vanishes at the AD point while the quark condensate remains finite.

Now let us work out the dyon condensate. More generally let us introduce the dyon field D_i , $i = 1, 2, 3$, which stands for charge, monopole and $(1, 1)$ dyon, $D_i = (Q_+, M, D)$. The arguments of the previous subsection which led us to the result (36) for monopole condensate gives for $\langle \tilde{D}_i \, D_i \rangle$

$$\langle \tilde{D}_i \, D_i \rangle = \frac{2\, i \, \zeta_i \, \mu}{\sqrt{-\det h}} \left(u_i^2 - \frac{3}{4} \, m \, \Lambda_1^3 \right)^{1/4} , \tag{48}$$

where u_i is the position of the i-th point in the u plane and the ζ_i are phase factors.

For the monopole condensate at real values of m larger than the $m_{\rm AD}$ Eq. (36) gives

$$\zeta_M = 1, \tag{49}$$

while for the charge condensate from Eq. (45) we have

$$\zeta_C = -i. \tag{50}$$

In fact one can fix the charge phase factor by imposing the condition that the charge condensate should approach the value $2m\mu$ in the large m limit. For the dyon the phase factor is

$$\zeta_D = i \, . \tag{51}$$

At the particular AD point we have chosen the monopole and charge condensates vanish, while the dyon condensate remains non-zero, see (48). Below the AD point, condensates are still given by Eq. (48), but the charge and monopole phase factors can change [1]. The dyon phase factor (51) does not change when we move through the AD point because the dyon condensate does not vanish at this point.

In the limit $m = 0$ we should recover the Z_3-symmetry for the values of condensates. From Eq. (48) it is clear that the absolute values of all three condensates are equal because the values of the three roots u_i are on the circle in the u plane, see (26). Imposing the requirement of Z_3 symmetry at $m = 0$ we can fix the unknown phase factors ζ_C and ζ_M below the AD point using the value (51) for dyon. This gives

$$\zeta_C = i \, , \quad \zeta_M = -i \, . \tag{52}$$

[1] Note that the quantum numbers of the "charge" and "monopole" are also transformed, see [28]

3.3 Photino and gaugino condensates

The gaugino condensate $\langle \mathrm{Tr}\, \lambda^2 \rangle$ we found in the previous section can be viewed as a sum of the condensates for charged gauginos and the photino,

$$\left\langle \mathrm{Tr}\, \lambda^2 \right\rangle = \left\langle \lambda^+ \lambda^- \right\rangle + \frac{1}{2} \left\langle \lambda^3 \lambda^3 \right\rangle \tag{53}$$

In gauge invariant form the photino condensate can be associated with

$$\left\langle (\mathrm{Tr}\, W \Phi)^2 \right\rangle \tag{54}$$

We argue here that the photino condensate vanishes so that the gaugino condensate is solely due to the charged gluino.

Let us start with the Coulomb branch in the $\mathcal{N}=2$ theory. All gaugino condensates vanish in $\mathcal{N}=2$ for a simple reason: λ^2 is *not* the lowest component in the corresponding $\mathcal{N}=2$ supermultiplet. When the perturbation $\mu\, U$ which breaks $\mathcal{N}=2$ is added to the superpotential the gaugino condensate is proportional to μ. However, the term $\mu\, U$ in the superpotential does not break $\mathcal{N}=2$ SUSY in the effective QED. Consider, for example, the monopole vacuum. The corresponding effective superpotential is given by Eq. (30), where in the expansion of U as function of A_D it is sufficient to retain only linear term. It was shown in [29] that the perturbation linear in A_D does not break $\mathcal{N}=2$ in the effective QED. An immediate consequence of this observation is that the photino condensate continues to vanish.

4 The Argyres-Douglas point: how well is the theory defined

As discussed in the Introduction, at the AD point we encounter the problem of not having a uniquely defined vacuum state. Indeed, when the mass parameter m approaches its AD value m_{AD} we deal with two vacuum states which can be distinguished by values of the chiral condensates. It is unlikely that the number of states with zero energy will change when we reach the AD point, it is very similar to the Witten index. However, the continuity of the chiral condensates we obtained above shows that they are no longer parameters which differentiate the two states once we reach the AD point.

This does not prove the absence of a relevant order parameter so the quest can be continued. A natural possibility to consider is a domain walls interpolating between colliding vacua. In the case of BPS domain walls their tension is given by the central charge [26],

$$T_{ab} = 2\, |\mathcal{W}_{\mathrm{eff}}(v_a) - \mathcal{W}_{\mathrm{eff}}(v_b)| \tag{55}$$

where a,b label the colliding vacua. The central charge here is expressed via values of exact superpotential (3) in corresponding vacua. The continuity of the condensate v

shows that the domain wall becomes tensionless at the AD point, $T \propto (m - m_{\rm AD})^{3/2}$ when $m \to m_{\rm AD}$. If such a domain wall were observable at the AD point it could serve as a signal of two vacua.

We argue, however, that this domain wall is not observable in continuum limit. The crucial point is that the wall is built out of massless fields, therefore its thickness is infinite at the AD point. This makes it impossible to observe this tensionless wall in any physical experiment of a limited spatial scale.

In the conclusion of this section let us review briefly the brane construction of $\mathcal{N}=1$ vacua. Gauge theories are realized on brane worldvolumes. Brane configurations responsible for $\mathcal{N}=1$ theories were suggested in [15] and a derivation of domain wall tensions from analysis of Riemann surfaces (which is similar to the calculation of the masses of BPS particles in $\mathcal{N}=2$ theories) can be found in [16]. The brane configuration for the $\mathcal{N}=1$ theory with one flavor and SU(2) gauge group is described by Riemann surface embedded into three dimensional complex space C^3 parametrized by three variables t, v and w. The embedding is given by the following equations

$$
\begin{aligned}
&v + m = \frac{(w - w_+)(w - w_-)}{\mu w}\,, \qquad && t = \mu^{-2} w (w - w_+)(w - w_-)\,; \\
&w_+ + \frac{1}{2}\, w_- + m\mu = 0\,, && w_-^2 w_+ = -(\mu\Lambda_1)^3\,.
\end{aligned} \tag{56}
$$

with free parameters μ, m, Λ. The tension of the walls, which have the interpretation of the M5 branes wrapping three-cycle with the boundaries on the Riemann surface above can be calculated by integrating the holomorphic three-form over this cycle

$$
T = \int \mathrm{d}v \wedge \mathrm{d}w \wedge \mathrm{d}(\log t) \tag{57}
$$

Let us consider the geometry of the brane configuration near the AD point. It was shown recently [30] that the AD point corresponds to a singular Calabi-Yau 3-manifold which is resolved if one adds particular perturbation. Since the tension is defined by integration of the holomorphic 3-form around the resolved singularity the tensionless wall has a geometrical interpretation as the M5 brane wrapping this vanishing cycle. Actually, the curves can be considered as fibered over the complex m plane and the AD singularities correspond to the appearance of vanishing cycles in the fiber in a manner quite similar to the Seiberg-Witten solution of $\mathcal{N}=2$ theories where vanishing cycles correspond to massless BPS particles.

5 Conclusions

The approach of this work is similar to that used in SQCD. Namely, we integrate out the adjoint field which leads, in some range of parameters, to an SQCD-like effective superpotential. This superpotential describes the low energy theory at weak coupling where we have full theoretical control. The nonperturbative part is given by the ADS superpotential generated at the one instanton level. The adjoint field shows up only as an extra (as compared with SQCD) nonrenormalizable term quartic in the fundamental fields.

Results for chiral condensates of matter and gaugino fields are continued into the range of a small adjoint mass where we find a complete matching with the $\mathcal{N}=2$ Seiberg-Witten solution. The Argyres-Douglas points introduced in $\mathcal{N}=2$ theories are shown to exists in the $\mathcal{N}=1$ theory as well. Although the bulk of our results for matter and gaugino condensates overlaps with what is known in the literature we think that our approach clarifies some aspects of duality in $\mathcal{N}=1$ theories.

We then analyze monopole, charge and dyon condensates departing from the Coulomb branch of the $\mathcal{N}=2$ theory. This resulted in explicit relations between these condensates and those of the fundamental matter. The most interesting phenomenon occurs at the AD point: when the monopole and charge vacua collide both the monopole and charge condensates vanish. We interpret this as a deconfinement of electric and magnetic charges at the AD point.

In our approach we see straightforwardly that the one-instanton generated ADS superpotential is the only nonperturbative input needed to fix all chiral condensates. The general nature of this statement is seen from our derivation which relates polynomial coefficients in the Seiberg-Witten curve to the ADS superpotential.

Let us mention a relation to finite-dimensional integrable systems. It was recognized that $\mathcal{N}=2$ theories are governed by finite-dimensional integrable systems. The integrable system responsible for $\mathcal{N}=2$ SQCD was identified with the nonhomogenious XXX spin chain [31]. After perturbation to the $\mathcal{N}=1$ theory the Hamiltonian of the integrable system is expected to coincide with the superpotential of corresponding $\mathcal{N}=1$ theory. This has been confirmed by direct calculation in the pure $\mathcal{N}=2$ gauge theory [32] as well in the theory with a massive adjoint multiplet [33]. It would be very interesting to find a similar connection between spin chain Hamiltonians and superpotentials in the $\mathcal{N}=1$ SQCD. One more point to be clarified is the meaning of the AD point within approach based on integrability. Since the quark mass is identified as a value of spin [31] one might expect that at particular spin values corresponding to the AD mass, the XXX spin chain would have additional symmetries similar to superconformal ones. We hope to discuss these points in more details elsewhere.

In this paper we considered only the SU(2) theory with one flavor postponing the generic N_c, N_f case for a separate publication. The most interesting problem in the generic situation involves Seiberg IR duality of the electric SU(N_c) theory with N_f flavors and the magnetic SU($N_f - N_c$) theory. In generic case of nondegenerate fundamental masses we expect deconfinement at the AD points. A degeneracy in fundamental masses leads to the appearance of Higgs branches. The approach of the present paper can be applied to this case as well. However, since Higgs branches do not disappear at the AD points [20] we do not expect deconfinement to occur in this case [34].

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